Causal EM for counterfactual inference, with an application to palliative care

(me)

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Artificialy

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IDSIA common
institute of USI & SUPSI
since 2000

SUPSI

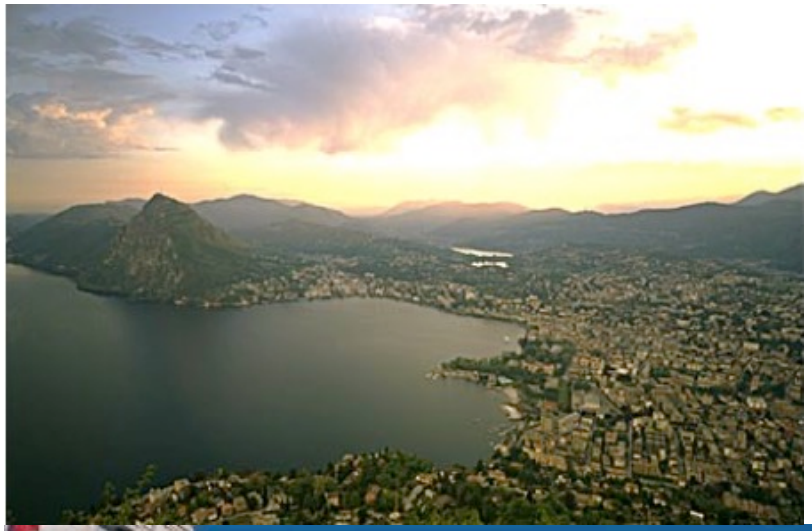
Istituto Dalle Molle di studi sull'intelligenza artificiale

IDSIA is a research institute on **Artificial Intelligence** founded in 1988 in Lugano, Switzerland

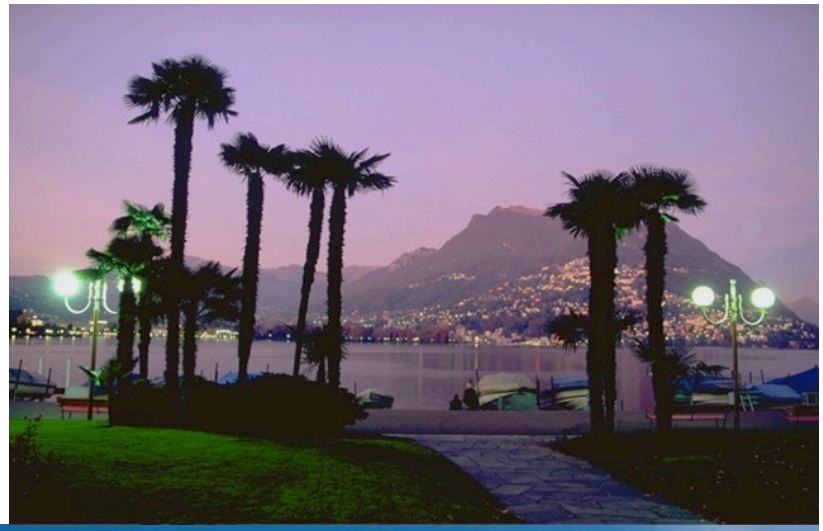
121 people = 16 Professors, 51 Researchers, 45 PhD students, 9 Research assistants

Thanks to Italian philantropist Angelo Dalle Molle (1908-2002)

«*The progress of science in general, and that of the emerging computer science in particular, should not subjugate people, but rather benefit them*»

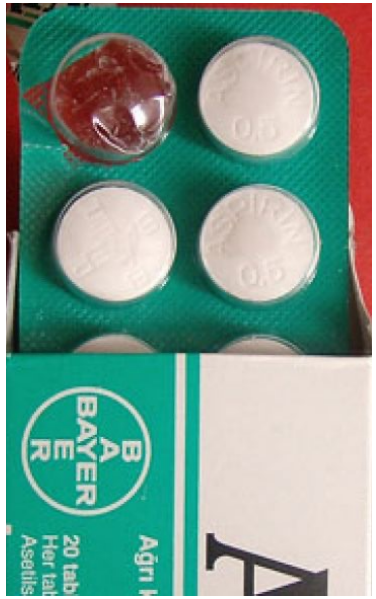


Lugano



Headache

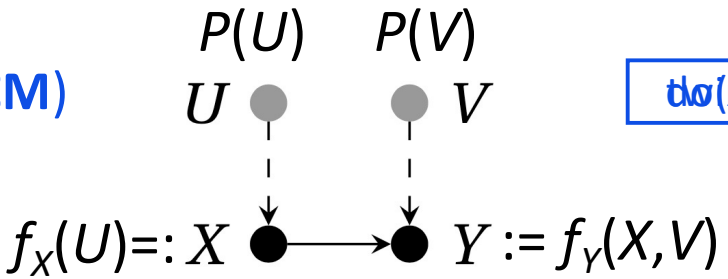
- You take an aspirin (X=1) and your headache vanishes (Y=1)



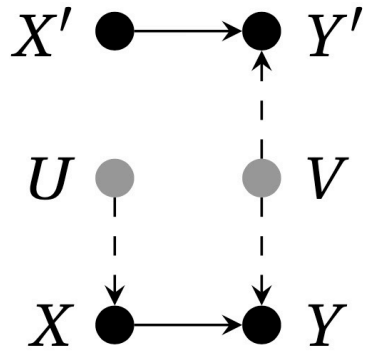
- What is the probability that this has been due to the aspirin?
  - Data says: $P(Y{=}0|X{=}0){=}0.5$ and $P(Y{=}0|X{=}1){=}0.1$

- What if I had **not** taken the aspirin, would have headache stayed?
- Probability of necessity (*PN*): $P(Y_{X=0} = 0|Y = 1, X = 1)$

- We need a fully specified structural causal model (**SCM**) to compute it
  - **What if not?**
  - **Yet *P*(*X*,*Y*) is available**



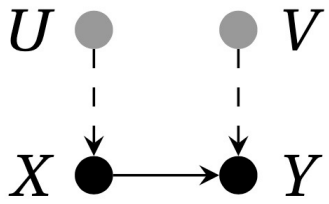
f_X, f_Y are so-called **structural equations**



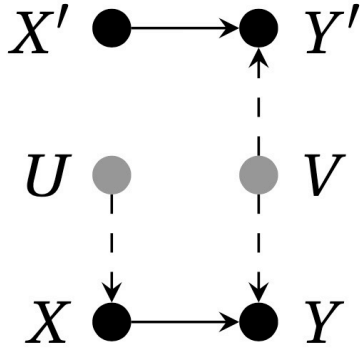
PN = $P(Y'|X'{=}0,X{=}1,Y{=}1)$

Causal inference
(via *credal nets* = sets of Bayesian nets)

- Select number of states for U and V
- No knowledge:
  - Conservative specification (canonical partition) |U| = 2, |V| = 4



- Deterministic functions written via 0-1 valued probabilities for P(X|U) and P(Y|X,V)
- Propagate P(X,Y) back to find out P(U) and P(V)
- We get P(U) = P(X) and P(V) = [t, 0.4 + t, 0.5 - t, 0.1 - t], t∈[0,0.1] (we call this a **credal set** = a set of distributions)
- Create its *twin net* where PN = P(Y'|X'=0,X=1,Y=1)

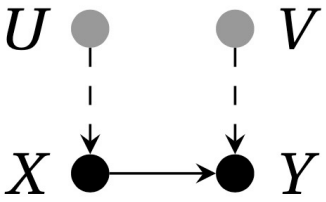


- Run an *exact* credal net algorithm to finally get 4/9 <= PN <= 5/9

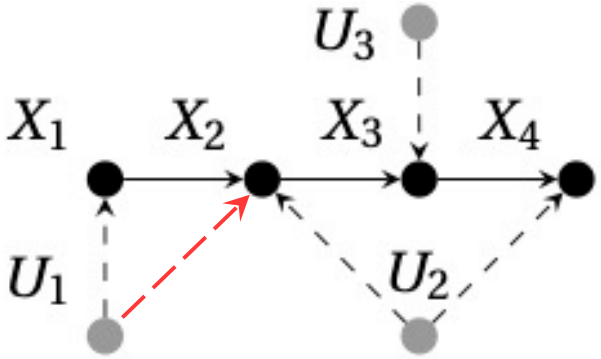
One can *in principle* solve all counterfactuals with this methodology

Problem

- The previous exact approach works well 😊
  with Markovian structural causal models (SCMs)



- So-so 🙂 with quasi-Markovian ones
- And does not 😕 for non-quasi-Markovian SCMs



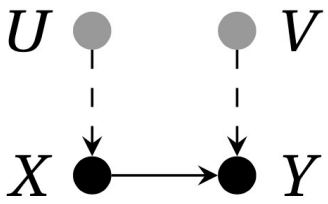
- Thm.: Causal inference (interventions) is NP-hard even in polytree-SCMs
  – Hardly surprising if you're in credal nets

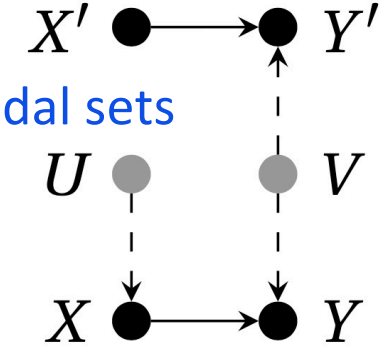
- Let's approximate!
- Idea:

the exogenous variables are missing at random ⇒ use the **EM**!

EM for Causal Computation (EMCC)

- Say we have a data set D of iid (x,y) instances
- Randomly initialise P(U) and P(V)
- Run the EM up to convergence





- Cor.: At convergence, P(U) and P(V) belong to their corresponding credal sets
  - EM samples the space of compatible fully specified SCMs!
  - On each of them, we can compute PN on its twin graph via Bayesian nets

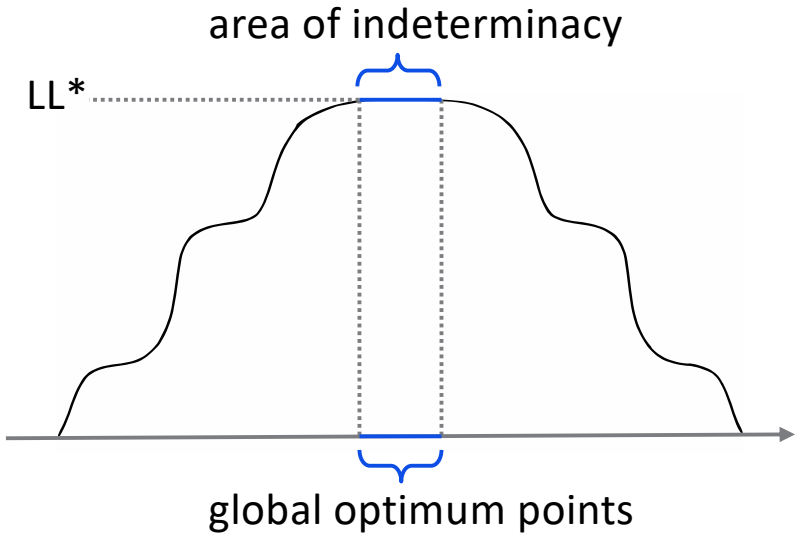
- Repeat k times: random initialisation + EM up to convergence + BN algorithm
  - You get $\{P_i(U),P_i(V)\}$ and PN_i, for i=1, …, k
  - A set of k points inside the interval [4/9,5/9]

- Take min and max and you get an **inner** approximation: $[a,b] \subseteq [4/9,5/9]$
  - k=20 already gives a pretty good approximation

works for any semi-Markovian SCM with categorical endogenous variables

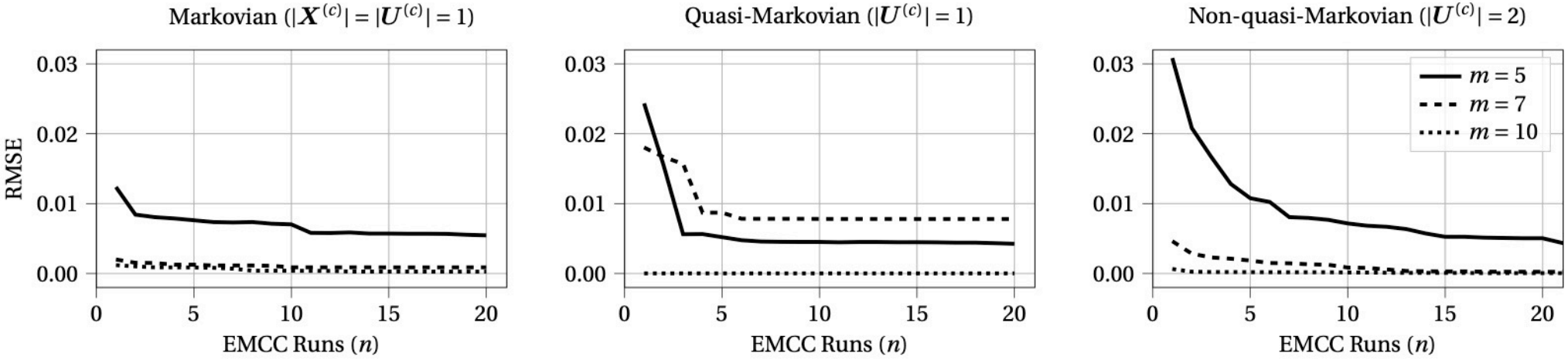
Why does it work?

- Thm.: The (log-)likelihood is unimodal
- Cor.: The global optimum points are IFF $P(U_1)$, …, $P(U_m)$ in their resp. credal sets



How well does it work?

(code available at github.com/idsia/credici)



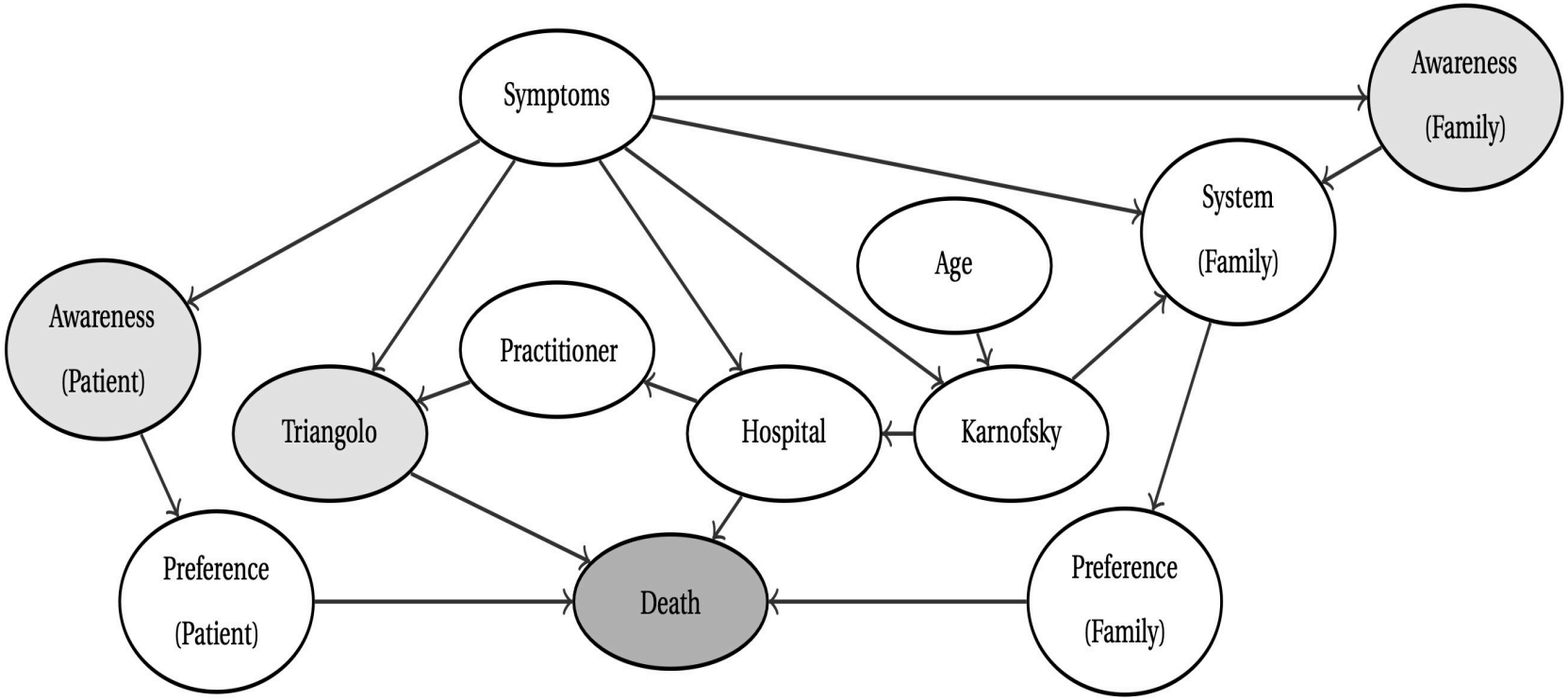
- Thm: $$P\left(a - \varepsilon L \leq a^* \leq b^* \leq b + \varepsilon L \,\middle|\, \rho\right) = \frac{\int_0^{\delta/2}\int_0^{\delta/2} P(x, y; L, \alpha, \beta, k)\,\mathrm{d}x\,\mathrm{d}y}{\int_0^{a+(1-b)}\int_0^{a+(1-b)-y} P(x, y; L, \alpha, \beta, k)\,\mathrm{d}x\,\mathrm{d}y}$$

$$P(x, y; L, \alpha, \beta, k) = \left(\frac{(L+x)^\alpha {}_2F_1(\alpha, 1-\beta, \alpha+1, \frac{L+x}{L+x+y}) - x^\alpha {}_2F_1(\alpha, 1-\beta, \alpha+1, \frac{x}{L+x+y})}{\alpha(L+x+y)^\alpha B(\alpha, \beta)}\right)^k$$

  - ${}_2F_1$ is the ordinary Gaussian hypergeometric function and *B* denotes a beta function
  - Remember [a,b] ⊆ [a*,b*]
  - k=20–30 runs already fairly good approximation

- **Corollary 3** *If $a = b$, i.e., all k runs in ρ are equal, then* $P(a^* = b^* | \rho) = 1 + 9/3^k - 8/2^k$
  - 9 equal runs => identifiable at 99% confidence

An application in palliative care

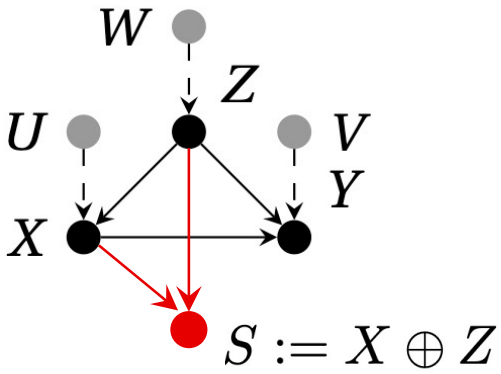


- Data from 116 patients about all the Boolean variables in the network => X_1, …, X_{12}
- No latent confounders => structural causal model is Markovian => U_i -> X_i (i=1, …,12)
  - Use the conservative specification
- Compute *PNS* := $P(Death_{X=yes} = \text{home}, Death_{X=no} = \text{hospital})$ w.r.t. the controllable X's
- EMCC: Triangolo 27−35%; family's awareness 4−11%; patient's awareness 3−11%
  - By the very Triangolo variable we can change the fate for ~30% of patients

EMCC extended to selection bias

- Consider (*X*,*Y*,*Z*) = (Treatment, Outcome, Gender) with SCM and data as shown

Treatment (X)	Recovery (Y)	Gender (Z)	#
0	0	0	2
1	0	0	41
0	1	0	114
1	1	0	313
0	0	1	107
1	0	1	109
0	1	1	13
1	1	1	1



- Treated males (*X*=1,*Z*=1) and untreated females (*X*=0,*Z*=0) systematically not reported
  - Case of **selection bias**
  - Can we still say something about the overall population?

EMCC extended to selection bias

- Consider (*X*,*Y*,*Z*) = (Treatment, Outcome, Gender) with SCM and data as shown

$\mathcal{D}$

Z	X	Y	S	#
0	0	0	0	2
0	0	1	0	114
0	1	0	1	41
0	1	1	1	313
1	0	0	1	107
1	0	1	1	13
1	1	0	0	109
1	1	1	0	1

$\mathcal{D}_1$

W	U	V	Z	X	Y	S	#
*	*	*	0	1	0	1	41
*	*	*	0	1	1	1	313
*	*	*	1	0	0	1	107
*	*	*	1	0	1	1	13

$\mathcal{D}_0$

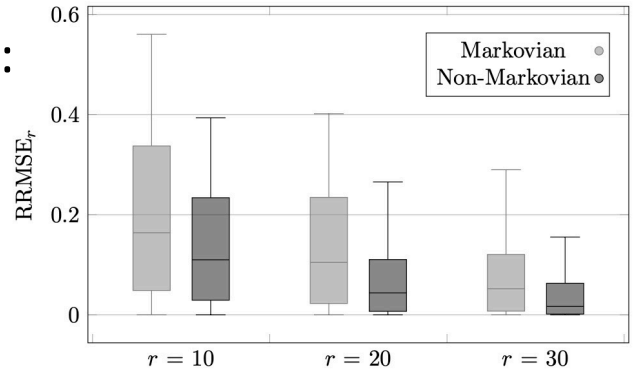
W	U	V	Z	X	Y	S	#
*	*	*	*	*	*	0	226

- EMCC is applied as before with the only difference that the iteration becomes

$$P_{t+1}(U) \leftarrow \frac{d_0 P_t(U|S=0) + \sum_{\boldsymbol{x}\in\mathcal{D}_1} P_t(U|\boldsymbol{x})}{(d_0 + d_1)}$$

- Main results hold as before:
  - Thm. 1: The (log-)likelihood is unimodal
  - Cor. 1: The global optimum points are IFF $P(U_1)$, …, $P(U_m)$ in their resp. credal sets

Experiments:



Conclusions

- The EMCC is based on simple ideas and tools
  - It should not be too difficult to join it to other models
  - Or to extend it to the continuous case
  - It might lead to `simple', while general, ways to join causal inference with machine learning

- It delivers guaranteed inner approximations
  - Outer ones are safer but tend to be more difficult to yield without becoming loose
  - Yet EMCC is anytime and can easily be made parallel
  - And we can yield credible intervals to increase safety with some guarantee

- More work is certainly needed on all these fronts

For now, we have automated counterfactual computation also under selection bias