

WHAT SHOULD BE THE FOCUS OF STATISTICAL PERSONALISED MEDICINE?

Philip Dawid
University of Cambridge

Joint work with Mats Stensrud, Aaron Sarvet and Stephen Senn

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OVERVIEW

- Scott Mueller and Judea Pearl have recently been promoting a new statistical approach to personalised medicine. In contrast to the traditional statistical approach based on decision theory, their idea is that medical decisions should focus on the “counterfactual contrast” between a patient’s actual response to the treatment they get, and the (necessarily unobservable) “potential response” to the treatment they do not get.
- Mueller and Pearl further claim that combining observational and experimental data can lead to improved decisions.
- We deconstruct and critique their arguments, and reaffirm the appropriateness of the traditional approach.

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Medical Decisions

L : background characteristics of patient

X : give treatment?

Y : response

- Data available on (L, X, Y) for many past patients
- How should doctor use L to decide X ?
– or to prioritize patients for treatment?

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BIG Assumption

L : background characteristics of patient

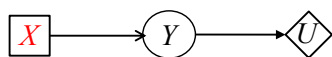
X : give treatment?

Y : response

- Data available on (L, X, Y) for many past patients
- *We assume that new patient is exchangeable with previous cases*
– typically not so for clinical trials

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Decision-Theoretic (DT) Approach



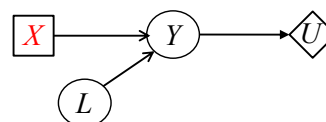
- L absent
- Value outcome $Y=y$ with utility $U = u(y)$
- From data, estimate distribution of Y for each hypothesised intervention $X \leftarrow x$
- Choose x to maximise expected utility

$$E\{U(Y) \mid X \leftarrow x\}$$

If Y is binary, maximise $P(Y = 1 \mid X \leftarrow x)$

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Use of Covariate Information

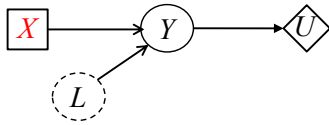


- **For current patient, can observe $L=l$**
- From data, can estimate conditional distributions of Y , given $L, X \leftarrow x$
– statistical/ML techniques
- Choose x to maximise “personalised” expected utility

$$E\{U(Y) \mid L=l, X \leftarrow x\}$$

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Use of Covariate Information

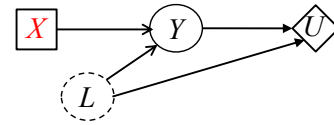


- For current patient, can't observe L
- From data, can estimate conditional distributions of Y , given L , $X \leftarrow x$
– but can't use this conditional distribution
- **Revert:** Still choose x to maximise overall expected utility

$$E\{U(Y) \mid X \leftarrow x\}$$

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Use of Covariate Information



- For current patient, can't observe L
- **But utility is now $U(L, Y)$**
- From data, estimate joint distribution of Y and L , given $X \leftarrow x$
- Choose x to maximise expected utility

$$E\{U(L, Y) \mid X \leftarrow x\}$$

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Mueller and Pearl (MP) Approach

- Introduce $Y(x)$, “potential response” to intervention $X \leftarrow x$
- Consider $(Y(0), Y(1))$ as having a joint distribution, unaffected by decision $X \leftarrow x$ taken
- Compare outcome $Y(1)$ if treated with outcome $Y(0)$ if not
 - $Y(1) = Y(0)$ – no effect
 - $Y(1) > Y(0)$ – benefit
 - $Y(1) < Y(0)$ – harm
- Treat accounting for expected benefit/harm

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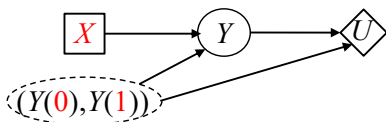
MP Approach

Three strands to their approach:

1. Assess value of treatment in terms of pair of potential outcomes $(Y(0), Y(1))$
e.g., $P(\text{benefit}), P(\text{harm})$
2. Use data to set bounds on their probabilities
3. Combine experimental and observational data

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1. Utility Function



- Assign utility U depending on both actual *and* counterfactual response (takes $L = (Y(0), Y(1))$)
 - e.g., 1 if benefit, -3 if harm, 0 if untreated
 - $P(\text{benefit}) - 3 \times P(\text{harm})$ vs. 0

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1. Utility Function

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 - e.g., 1 if benefit, -3 if harm, 0 if untreated
 - $P(\text{benefit}) - 3 \times P(\text{harm})$ vs. 0

BUT—

- Impossible ever to observe both $Y(0)$ and $Y(1)$ together
- So typically *can not identify joint distribution* of $(Y(0), Y(1))$ from data
– – likewise, $P(\text{benefit}), P(\text{harm})$

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2. Probability Bounds

Can not identify joint distribution of $(Y(0), Y(1))$ from data

HOWEVER:

From *experimental data*, yielding the marginal distributions of each of $Y(0)$, $Y(1)$ separately, we can *set bounds* on their joint distribution (and so on $P(\text{benefit})$, $P(\text{harm}), \dots$)

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2. Probability Bounds

- From *experimental data*, yielding the marginal distributions of each of $Y(0)$, $Y(1)$ separately, we can *set bounds* on their joint distribution (and so on $P(\text{benefit})$, $P(\text{harm}), \dots$)

- $P(Y=1 | X \leftarrow 1) = 0.49$
- $P(Y=1 | X \leftarrow 0) = 0.21$

$$0 \leq x \leq .21$$

		Y(1)		
		1	0	
Y(0)	1	x	$.21 - x$.21
	0	$.49 - x$	$.3 + x$.79
		.49	.51	1

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2. Probability Bounds

- From *experimental data*, yielding the *marginal* distributions of each of $Y(0)$, $Y(1)$ separately, we can *set bounds* on their *joint* distribution (and so on $P(\text{benefit})$, $P(\text{harm}), \dots$)
- This joint distribution is *fully identified* just when *either* $Y(0)$ *or* $Y(1)$ is degenerate (at 0 or 1)
 - In this case either $P(\text{benefit}) = 0$, or $P(\text{harm}) = 0$
- Then clear what to do
- DT and MP agree

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Use of Covariate

- From experimental data, yielding the marginal distributions of each of $Y(0)$, $Y(1)$ separately, we can set bounds on their joint distribution (and so on $P(\text{benefit})$, $P(\text{harm}), \dots$)
- We can *improve* these bounds if there is a pre-treatment covariate L such that we know the joint distribution of (L, Y) under either intervention
 - even if we don't observe L for current patient
- Now we have full identification just when, *given* any value of L , either $Y(0)$ or $Y(1)$ is degenerate
- DM and MP agree if L observed

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3. Combining experimental and observational data

- One possible covariate is the *intention to treat* (ITT) variable X^* , which determines the *actual* treatment X in the observational case, but is overridden by the *applied* X in an experiment
- X^* sometimes observable in an experiment
- But even if not, by combining experimental and observational data we can still compute the experimental joint distributions of (X^*, Y)
- In either case this gives improved MP bounds

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Another BIG Assumption

- From *experimental data*, yielding the marginal distributions of each of $Y(0)$, $Y(1)$ separately, we can *set bounds* on their joint distribution (and so on $P(\text{benefit})$, $P(\text{harm}), \dots$)
- If we also have *observational data*, we can improve these bounds
- We must assume observational cases are exchangeable with experimental cases
 - typically not so for clinical trials

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EXAMPLE 1

- *Experimental data:*

$$P(\text{survive} \mid \text{drug}) = 0.49, P(\text{survive} \mid \text{no drug}) = 0.21$$

DT: Better to give treatment

MP: We have

$$\begin{aligned} 0.28 &\leq P(\text{benefit}) \leq 0.49 \\ 0 &\leq P(\text{harm}) \leq 0.21 \end{aligned}$$

What to do?

- Adding *Observational data:*

$P(\text{drug}) = 0.7, P(\text{survive} \mid \text{drug}) = 0.27, P(\text{survive} \mid \text{no drug}) = 0.7$
we get

$$\begin{aligned} P(\text{benefit}) &= 0.28 \\ P(\text{harm}) &= 0 \\ \text{—so give treatment} \end{aligned}$$

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EXAMPLE 2

- *Experimental data:*

$$P(\text{survive} \mid \text{drug}) = 0.49, P(\text{survive} \mid \text{no drug}) = 0.21$$

DT: Better to give treatment

MP: We have

$$\begin{aligned} 0.28 &\leq P(\text{benefit}) \leq 0.49 \\ 0 &\leq P(\text{harm}) \leq 0.21 \end{aligned}$$

What to do?

- Adding *Observational data:*

$P(\text{drug}) = 0.7, P(\text{survive} \mid \text{drug}) = 0.7, P(\text{survive} \mid \text{no drug}) = 0.7$
we get

$$\begin{aligned} P(\text{benefit}) &= 0.49 \\ P(\text{harm}) &= 0.21 \\ 0.49 - 3 \times 0.21 &< 0. \text{ So don't treat.} \\ \text{different from DT because of utility function} \end{aligned}$$

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EXAMPLE 2

- *Experimental data:*

$$P(\text{survive} \mid \text{drug}) = 0.49, P(\text{survive} \mid \text{no drug}) = 0.21$$

DT: Better to give treatment

MP:

$$\begin{aligned} P(\text{benefit}) &= 0.49 \\ P(\text{harm}) &= 0.21 \\ 0.49 - 3 \times 0.21 &< 0. \text{ So don't treat.} \end{aligned}$$

In a population of similar individuals, treating everyone (DT) would lead to 49% surviving, whereas treating no one (MP) would lead to 21% surviving!

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EXAMPLE 2

- *Experimental data:*

$$P(\text{survive} \mid \text{drug}) = 0.49, P(\text{survive} \mid \text{no drug}) = 0.21$$

DT: Better to give treatment

MP:

$$\begin{aligned} P(\text{benefit}) &= 0.49 \\ P(\text{harm}) &= 0.21 \\ 0.49 - 3 \times 0.21 &< 0. \text{ So don't treat.} \end{aligned}$$

Alternatively the utilities for benefit and harm imply the following “marginal utilities”—

$$\begin{aligned} U(\text{treat}, Y=1) &= 1. \quad U(\text{treat}, Y=0) = -1.24. \quad E\{U(\text{treat}, Y)\} = -0.14 \\ U(\text{not treat}, Y=1) &= U(\text{not treat}, Y=0) = 0. \quad E\{U(\text{not treat}, Y)\} = 0 \end{aligned}$$

So don't treat

But why should the utility of the outcome depend on whether or not treated?

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EXAMPLE 3

- *Experimental data:*

$$P(\text{survive} \mid \text{drug}) = 0.49, P(\text{survive} \mid \text{no drug}) = 0.21$$

DT: Better to give treatment

MP: We have

$$\begin{aligned} 0.28 &\leq P(\text{benefit}) \leq 0.49 \\ 0 &\leq P(\text{harm}) \leq 0.21 \end{aligned}$$

What to do?

- Adding *Observational data:*

$P(\text{drug}) = 0.7, P(\text{survive} \mid \text{drug}) = 0.29, P(\text{survive} \mid \text{no drug}) = 0.33$
we get

$$\begin{aligned} 0.28 &\leq P(\text{benefit}) \leq 0.40 \\ 0 &\leq P(\text{harm}) \leq 0.12 \end{aligned}$$

What to do?

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3. Combining experimental and observational data

- In Examples 1 & 2, after combining data, the joint distribution of $(Y(0), Y(1))$ was fully identified
- This happens just when, for either *intended treatment* $X^* = x^*$, there is a setting $X \leftarrow x^\dagger$ under which Y is determined ($= 0$ or 1)
- Ideally, observe x^* for new patient, and choose $X \leftarrow x^\dagger$ if that yields $Y = 1$; else, other setting
– DT and MP agree
- More generally, observe X^* to improve DT

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3. Combining experimental and observational data

- In **Example 1**, after combining data, the joint distribution of $(Y(0), Y(1))$ was fully identified
 - $P(\text{benefit}) = 0.29, P(\text{harm}) = 0$
- We find:
 - For $X^* = 0, X \leftarrow 1 \Rightarrow Y = 1$. So treat
 - For $X^* = 1, X \leftarrow 0 \Rightarrow Y = 0$. So treat
- So treat anyway! DT and MP agree

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3. Combining experimental and observational data

- In **Example 2**, after combining data, the joint distribution of $(Y(0), Y(1))$ was fully identified
 - $P(\text{benefit}) = 0.49, P(\text{harm}) = 0.21$
- We find:
 - For $X^* = 0, X \leftarrow 1 \Rightarrow Y = 0$. So don't treat
 - For $X^* = 1, X \leftarrow 0 \Rightarrow Y = 0$. So treat
- Helps if we can observe X^*

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3. Combining experimental and observational data

In **Example 3**, after combining data, the joint distribution of $(Y(0), Y(1))$ was *not* fully identified:
 $0.28 \leq P(\text{benefit}) \leq 0.40, 0 \leq P(\text{harm}) \leq 0.12$

- In this case we find:
 - $P(Y=1 | X^*=0, X \leftarrow 1) = 0.97$
 - $P(Y=1 | X^*=0, X \leftarrow 0) = 0.33$
 - So treat if $X^*=0$
 - $P(Y=1 | X^*=1, X \leftarrow 1) = 0.29$
 - $P(Y=1 | X^*=1, X \leftarrow 0) = 0.16$
 - So treat if $X^*=1$
- DT: Treat anyway (X^* wasn't helpful)

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Messages?

- Reason for combining observational and experimental data is to give us a handle on unobserved ITT variable X^*
- If we can observe X^* in a new patient, this can improve decision-making
- If we can't, too bad
 - *unless* utility depends on unobserved features
 - as it does for MP
- Is such a utility reasonable?

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